FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION-2023 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE:
- **(i)** Attempt **ONLY FIVE** questions. **ALL** questions carry **EQUAL** marks
- **(ii)** All the parts (if any) of each Question must be attempted at one place instead of at different places.
- **(iii)** Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
- **(iv)** No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- **(v)** Extra attempt of any question or any part of the attempted question will not be considered.
- **(vi)** **Use of Calculator is allowed**.

Q. No. 1. **(a)** Forces of magnitudes P, 2P, 3P, 4P act respectively along the sides AB, BC, CD, DA of a square ABCD, of side a and forces each of magnitude $(8\sqrt{2})$ P act along the diagonals BD, AC. Find the magnitude of the resultant force and the distance of its line of action from A. **(10)**

(b) A uniform rod AB of length a and weight W is freely hinged to a vertical wall at A and is maintained in equilibrium by a light string of length a fastened to B and to a point C at a distance b vertically above A. Prove that the reaction at the hinge A is **(10)**

$$W\frac{\sqrt{(a^2+2b^2)}}{2b}$$

and find the tension in the string.

Q. No. 2. **(a)** Use Runge-Kutta method of order two to solve the following differential equation at x=1.2 by taking h=0.1 **(10)**

$$\frac{dy}{dx}=\frac{3x+y}{x+2y} \quad y(1)=1\,.$$

(b) Find the first and second derivatives of $f(x)$ at $x = 3$ from the following data using Newton's forward difference interpolation formula **(10)**

x	3	3.5	4	4.5	5	5.5
$f(x)$	4.1023	5.1047	8.1971	9.1096	4.1122	6.1148

Q. No. 3. **(a)** Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2,-1,2)$. **(8)**

(b) Show that **(6)**

$$\nabla r^n = nr^{n-2}r$$

(c) Find the total work done in a moving particle in a force field given by $F=3xy\,\boldsymbol{i}-5\,z\,\boldsymbol{j}+10\,x\,\boldsymbol{k}$ along the curve $x=t^2+1, y=2t^2, z=t^3$ from $t=1$ to $t=2$. **(6)**

Q. No.4. **(a)** A particle P moves in a plane in such a way that at any time t, its distance from a fixed point O is $r=a\,t+b\,t^2$ and the line connecting O and P makes an angle $\theta=ct^{\frac{3}{2}}$ with a fixed line OA. Find the radial and transverse components of the velocity and acceleration of the particle at $t=1$. **(10)**

(b) Solve the following Bernouli's equation **(10)**

$$x\frac{dy}{dx}+y=\frac{1}{y^2}$$

Q. No. 5. **(a)** Solve the following differential equation **(10)**

$$x\,dy=(x\,sinx-y)\,dx$$

(b) Find the general solution of the higher order differential equation **(10)**

$$y'''+8y''=-6\,x^2+9x+2$$

Q. No. 6. **(a)** Find solution of $4y'' + y = 0$ in the form of power series in x. **(10)**

(b) Solve the following differential equation by variation of parameters **(10)**

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

Q. No. 7. **(a)** Find real root of the equation $2x - 3\sin(x) - 5 = 0$ up to 4 decimal places by secant method. **(10)**

(b) Solve the following system of equations by Guass Seidel method. Perform only five iterations. **(10)**

$$8x_1 - x_2 - x_3 = 6$$
$$x_1 + 6x_2 + x_3 = 8$$
$$x_1 - x_2 + 5x_3 = 5$$

Q. No. 8. **(a)** Expand $f(x) = \sin x,\ 0 < x < \pi,$ in a Fourier cosine series. **(10)**

(b) Use the method of separation of variables to find the solution of the following boundary value problem **(10)**

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad 0 \le x \le a, \qquad 0 \le y \le b$$
$$u_x(0,y) = 0, \quad u_x(a,y) = 0,$$
$$u_y(x,b) = 0, \quad u(x,0) = f(x).$$

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