FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION-2021 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE:
- **(i)** Attempt **ONLY FIVE** questions. **ALL** questions carry **EQUAL** marks
- **(ii)** All the parts (if any) of each Question must be attempted at one place instead of at different places.
- **(iii)** Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
- **(iv)** No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- **(v)** Extra attempt of any question or any part of the attempted question will not be considered.
- **(vi)** **Use of Calculator is allowed**.

Q. No. 1. **(a)** Evaluate the surface integral $\iint \vec{A}.\vec{n}dS$ where $\vec{A} = z\vec{i} + x\vec{j} - 3y^2z\vec{k}$ and S is the portion of the cylinder $x^2 + y^2 = 8$ lying in the first octant between $z = 0$ and $z = 4$. **(10)**

(b) Prove that **(10)**

$$\nabla\big(f(r)\big) = \frac{f'(r)}{r}\vec{r},$$

where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$.

Q. No. 2. **(a)** The greatest resultant that two forces can have is of magnitude P and the least is of magnitude Q.Show that, when they act at an angle α, their resultant is of magnitude $\sqrt{P^2cos^2\frac{\alpha}{2} + Q^2sin^2\frac{\alpha}{2}}$. **(10)**

(b) A sphere of weight W and radius a is suspended by a string of length l from a point P and a weight w is also suspended from P by a string sufficiently long for the weight to hang below the sphere. Show that the inclination of the first string to the vertical is **(10)**

$$\sin^{-1}\frac{wa}{(W+w)(a+l)}.$$

Q. No. 3. **(a)** Show that the law of force towards the pole, of a particle describing the curve $r^n = a^n \cos n\theta$ is given by **(10)**

$$f = \frac{(n+1)h^2a^{2n}}{r^{2n+3}}.$$

(b) The maximum velocity that a particle executing simple harmonic motion of amplitude a attains, is v. If it is disturbed in such a way that its maximum velocity becomes nv. Find the change in the amplitude and the time-period of motion. **(10)**

Q. No.4. **(a)** Define ordinary and singular points of the differential equation $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$. When a singular point is said to be regular and irregular? Find regular and irregular singular points of the differential equation $(x^2-4)^2y'' + (x-2)y' + y = 0$. **(10)**

(b) Show that **(10)**

$$J_{3/2} = \sqrt{\frac{2}{\pi x}}\left[\frac{\sin x}{x} - \cos x\right].$$

Q. No. 5. **(a)** Solve the equation by using method of undetermined coefficients **(10)**

$$y'' - y' + y = 2\cos 3x.$$

(b) Use the method of Frobenius to find two linear independent series solutions in powers of x of the DE. **(10)**

$$x^2y'' - (x^2+x)y' + y = 0.$$

Q. No. 6. **(a)** Classify general second order partial differential equation (PDE) into elliptic, parabolic and hyperbolic form. Discuss the nature of the PDE $(1-x^2)u_{xx} - 2xyu_{xy} + (1-y^2)u_{yy} = 0$ at each $(x, y) \in R^2$. **(10)**

(b) Use the method of separation of variables to find the solution $u(x,t): [0,T] \times [0,L] \to R$ to the initial/boundary value problem **(10)**

$u_t(x,t) = u_{xx}(x,t)$ $\quad$ $for \quad 0 < t \leq T \; and \; 0 \leq x \leq L$,

$u(x,0) = f(x),$ $\quad$ $for \quad 0 \leq x \leq L,$

$u(0,t) = u(L,t) = 0,$ $\quad$ $for \quad 0 < t \leq T,$

where $f: [0,L] \to R$ is a known function.

Q. No. 7. **(a)** Use Simpson's 3/8 rule to estimate the integral **(10)**

$$\int_1^3 (x^3 - 2x^2 + 7x - 5)dx.$$

By comparing your answer with exact value, find the error.

(b) Solve the system of equations by Jacobi iterative method. **(10)**

$10x + 3y + z = 19, \quad 3x + 10y + 2z = 29, \quad x + 2y + 10z = 35$

Q. No. 8. **(a)** In the following table values of $y = x + \sin x^2$ are tabulated **(10)**

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
$f(x)$	1.84147	2.03562	2.19146	2.29290	2.32521	2.27807	2.14935

Construct a difference table and estimate $f(1.04)$ and $f(1.57)$.

(b) Use trapezoidal and Simpson's 1/3 rules to approximate $\int_0^{\pi/2} sin^2(x)dx$. Find a maximum bound for the error in each case. Compare your approximations with the actual result. **(10)**

****