MATHEMATICS HSSC–II
SECTION – A (Marks 20)

Time allowed: 25 Minutes

| Version Number | 4 | 1 | 1 | 2 |

Note: Section – A is compulsory. All parts of this section are to be answered on the separately provided OMR Answer Sheet which should be completed in the first 25 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q. 1 Choose the correct answer A / B / C / D by filling the relevant bubble for each question on the OMR Answer Sheet according to the instructions given there. Each part carries one mark.

1) If $\underline{a}=[1,2,-1]$, $\underline{b}=[1,-2,3]$ and $\underline{c}=[1,-7,-4]$ then $(\underline{a}\cdot\underline{b})\times\underline{c}$ is:

   A.   38   B.   Meaningless   C.   48   D.   20

2) $\lim\limits_{x\to\infty}\dfrac{x+e}{x-e}$ is equal to:

   A.   0   B. ∞ C.   1   D. -1

3) Which of the following equations represents an **odd** function?

   A. $f(x)=(x+2)^2$ B. $f(x)=\dfrac{3x}{x^2+1}$

   C. $f(x)=3x^4-2x^2+7$ D. $f(x)=\sin x+\cos x$

4) If $f(x)=(-x+9)^3$ and $g(x)=6$ then which of the following represents $g[f(x)]$

   A. -27 B. $(-x+9)^3$ C.   6   D.   27

5) What is the value of $\sqrt{1-x^2}\,\dfrac{d}{dx}(\sin^{-1}x+\cos^{-1}x)$

   A.   0   B. $\sqrt{1-x^2}$ C.   2   D. $\dfrac{1}{x}$

6) If $f(x+h)=a^{x+h}$ then $f'(x)$ is equal to:

   A. $a^{x+h}\ln(x+h)$ B. $a^x\ln a$ C. $a^x\ln x$ D. $a^{x+h}\ln a$

7) What is the derivative of $\dfrac{x^3-x}{x+1}$ with respect to x

   A. $1-2x$ B. $\dfrac{2x^3+3x^2-2x-1}{(x+1)^2}$

   C. $\dfrac{2x^3+3x^2-1}{x+1}$ D. $2x-1$

8) What evaluates $\displaystyle\int_0^1\sqrt{x^2-2x+1}\,dx$

   A.   1   B. $\dfrac{1}{2}$ C. $-\dfrac{1}{2}$ D. -1

9) Which of the following is an evaluated value of $\displaystyle\int_0^e\dfrac{|x|}{x}\,dx$

   A.   Undefined   B. e C. $\pm e$ D.   1

10) What is the value of dy of the function $f(x) = x^2$, when $x = 2$ and $dx = 0.01$?

A.   4   B.   0.4   C.   0.04   D.   0.02

11) What is the perpendicular distance between the line $x + y = 1$ and a point $\left(\dfrac{8}{3}, \dfrac{-5}{3}\right)$

A.   1   B.   0   C. $\dfrac{1}{\sqrt{2}}$ D. $\dfrac{1}{2}$

12) At what angle lines $3y = 2x + 5$ and $3x + 2y = 8$ cut each other?

A. $\dfrac{\pi}{2}$ B. $0°$ C. $\dfrac{\pi}{6}$ D. $\dfrac{\pi}{4}$

13) What is the slope of a line perpendicular to $3x - 4y + k = 0$

A. -1 B. $\dfrac{4}{3}$ C. $-\dfrac{4}{3}$ D. $\dfrac{-3}{4}$

14) $(0,0)$ is **NOT** a solution of which of the following inequalities?

A. $x - y < 1$ B. $2x + y < 1$ C. $-2x + y + 1 > 0$ D. $-2x + y < -1$

15) What is the length of the Latus rectum of a parabola $8y^2 = -32x$

A.   16   B.   4   C. -4 D.   8

16) Which of the following represents equations of asymptotes of hyperbola $\dfrac{x^2}{7^2} - \dfrac{y^2}{4^2} = 1$?

A. $x = \pm\dfrac{7}{4}y$ B. $x = \pm\dfrac{4}{7}y$ C. $y = \pm\dfrac{4}{7}x$ D. $y = \pm\dfrac{7}{4}x$

17) What is the eccentricity of a point circle $x^2 + y^2 = 0$?

A. $\dfrac{1}{\sqrt{2}}$ B.   1   C. $\sqrt{2}$ D.   0

18) What is the length of major axis of an ellipse $\dfrac{(x-1)^2}{2^2} + \dfrac{(y+1)^2}{3^2} = 1$

A.   18   B.   4   C.   6   D.   8

19) What is the volume of a parallelepiped, if its conterminous edges are $[2,-4,5], [2,-3,6]$ and $[0,-1,-1]$?

A.   15   B.   24   C.   16   D.   0

20) For what value of p , $[2,p,5]$ is perpendicular to $[3,1,p]$?

A. $\dfrac{2}{3}$ B. -1 C.   1   D. $\sqrt{5}$

MATHEMATICS HSSC–II

Time allowed: 2:35 Hours　　　　　　　　　**Total Marks Sections B and C: 80**

NOTE: Attempt any ten parts from Section 'B' and any five questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Graph paper will be provided on Demand.

SECTION – B (Marks 40)

Q. 2   Attempt any TEN parts. All parts carry equal marks.　　　　　　(10 x 4 = 40)

(i)　　Evaluate $\lim\limits_{\theta \to 0} \dfrac{\tan \theta - \sin \theta}{\sin^3 \theta}$

(ii)　　Discuss the continuity of $f(x)$ at $x = 3$, when $f(x) = \begin{cases} x - 1 & \text{if } x < 3 \\ 2x + 1 & \text{if } x \ge 3 \end{cases}$

(iii)　　Find $\dfrac{dy}{dx}$ if $x\sqrt{1+y} + y\sqrt{1+x} = 0$

(iv)　　Differentiate $\sin^3 x$ w.r.t $\cos^2 x$

(v)　　Show that $2^{x+h} = 2^x \left\{ 1 + (\ln 2)h + \dfrac{(\ln 2)^2}{\underline{|2}}h^2 + \dfrac{(\ln 2)^3}{\underline{|3}}h^3 + \ldots\ldots \right\}$

(vi)　　Evaluate $\displaystyle\int \dfrac{dx}{\left(1+x^2\right)^{\frac{3}{2}}}$

(vii)　　Evaluate $\displaystyle\int \dfrac{e^x(1+\sin x)}{1+\cos x}dx$

(viii)　　Find the points trisecting the join of $A(-1,4)$ and $B(6,2)$.

(ix)　　Find a joint equation of the straight lines through the origin and perpendicular to the lines represented by $x^2 + xy - 6y^2 = 0$

(x)　　Find equations of the tangents drawn from $(0,5)$ to $x^2 + y^2 = 16$.

(xi)　　A parabolic arch has a $100m$ base and height $25m$. Find the height of the arch at a point $30m$ from the centre of the base.

(xii)　　Find foci, vertices, and directrices of the ellipse $9x^2 + y^2 = 18$

(xiii)　　Find area of the parallelogram whose vertices are $A(1,2,-1)$, $B(4,2,-3)$, $C(6,-5,2)$ and $D(9,-5,0)$

(xiv)　　Prove that the points whose position vectors are $A(-6\underline{i}+3\underline{j}+2\underline{k})$, $B(3\underline{i}-2\underline{j}+4\underline{k})$, $C(5\underline{i}+7\underline{j}+3\underline{k})$ and $D(-13\underline{i}+17\underline{j}-\underline{k})$ are coplanar.

SECTION – C (Marks 40)

Note:　　Attempt any FIVE questions. All questions carry equal marks.　　　　(5 x 8 = 40)

Q. 3　　If θ is measured in radian, then prove that $\lim\limits_{\theta \to 0} \dfrac{\sin \theta}{\theta} = 1$

Q. 4　　If $x = a(\theta + \sin\theta)$, $y = a(1+\cos\theta)$, then show that $y^2\dfrac{d^2 y}{dx^2} + a = 0$

Q. 5　　Evaluate $\displaystyle\int e^x \sin 2x \cos x\, dx$

Q. 6　　The vertices of a triangle are $A(-2,3)$, $B(-4,1)$ and $C(3,5)$. Find the coordinates of the orthocentre of the triangle.

Q. 7　　Maximize the function defined as $f(x,y) = 2x + 3y$ subject to the constraints

　　　$2x + y \le 8$;　$x + 2y \le 14$;　$x \ge 0$; $y \ge 0$

Q. 8　　Find the equation of a circle passing through $A(5,1)$ and tangent to the line $2x - y - 10 = 0$ at $B(3,-4)$

Q. 9　　Show that midpoint of hypotenuse of a right triangle is equidistant from its vertices.

MATHEMATICS HSSC–II
SECTION – A (Marks 20)

Time allowed: 25 Minutes | **Version Number** | 4 | 1 | 1 | 6 |

Note: Section – A is compulsory. All parts of this section are to be answered on the separately provided OMR Answer Sheet which should be completed in the first 25 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q. 1 Choose the correct answer A / B / C / D by filling the relevant bubble for each question on the OMR Answer Sheet according to the instructions given there. Each part carries one mark.

1) Which of the following vectors is perpendicular to $[2,-1,1]$

    A. $[-1,6,8]$ B. $[-3,6,0]$ C. $[-1,-6,8]$ D. $[1,-6,8]$

2) $\lim\limits_{x \to 99} (100-x)^{\tan\frac{\pi}{2}}$ is equal to:

    A. 0 B. ∞ C. 99 D. 1

3) Which of the following represents domain of a real function $f(x) = \sqrt{1-x}.\ln x$

    A. $(-\infty, 0]$ B. $[0,1)$ C. $(0,1]$ D. $(0,+\infty)$

4) What is the maximum value of $|7\cos x - 5|$ for $x \in \mathbb{R}$?

    A. π B. 7 C. $\dfrac{\pi}{2}$ D. 2

5) If $f(x) = \sin x.g(x)$ with $g\left(\dfrac{\pi}{2}\right) = e$ and $g'\left(\dfrac{\pi}{2}\right) = \ln e$ then $f'\left(\dfrac{\pi}{2}\right) = ?$

    A. $e+1$ B. 0 C. 1 D. e

6) For $f(x) = \sinh^{-1}\left(\dfrac{x}{2}\right)$ which of the following is $f'(x)$

    A. $\dfrac{2}{\sqrt{x^2-4}}$ B. $\dfrac{1}{\sqrt{4+x^2}}$ C. $\dfrac{2}{\sqrt{4+x^2}}$ D. $\dfrac{1}{\sqrt{x^2-4}}$

7) If $f'(x) = -f(x)$ then which of the following represents $f(x)$

    A. $\dfrac{1}{2}e^{-2x+2}$ B. e^{1-x} C. e^{-1+x} D. e^{-x^2}

8) Evaluate $\int_0^{\frac{\pi}{4}} e^{\tan x} \cdot \sec^2 x\, dx$

    A. $e-1$ B. e C. $e+1$ D. 0

9) The result of $\int_{-\infty}^{0} \dfrac{e^x}{e^x+3}\, dx$ is:

    A. Undefined   B. $\ln\dfrac{4}{3}$ C. $\ln\dfrac{3}{4}$ D. 0

10) For what value of K , $\int_{-1}^{0}(3x^2+2x+K)dx = 5$

    A. 5 B. -5 C. -3 D. 0

11) Which of the following represents equation of a line passing through $(-8,5)$ and slope undefined?

    A. $x+8=0$ B. $y+5=0$ C. $x-8=0$ D. $y-5=0$

12) Which of the following lines passes through points $(-2,1)$ and $(6,-4)$

    A. $5x+8y-18=0$ B. $5x+8y-2=0$

    C. $5x+8y+2=0$ D. $5x-8y+18=0$

13) For what value of p, $3x+6y+8=0$ is perpendicular to $px+3y+7=0$

    A. 6 B. $\dfrac{1}{6}$ C. $-\dfrac{1}{6}$ D. -6

14) $(-1,-1)$ is a solution of which of the following inequalities?

    A. $-4x+3y<0$ B. $-x-2y<0$ C. $2x+y<-1$ D. $2x-y>10$

15) What is the length of the tangent drawn from $(-1,2)$ to the circle $x^2+y^2+4x+2y=0$?

    A. $\sqrt{5}$ B. 5 C. $\sqrt{13}$ D. $\sqrt{11}$

16) What is the length of the Latus rectum of an ellipse $\dfrac{x^2}{20^2}+\dfrac{y^2}{10^2}=1$

    A. 100 B. 0.50 C. 5 D. 10

17) Which of the following is an equation of a parabola with focus $(0,5)$ and vertex $(0,0)$

    A. $y^2=-20x$ B. $x^2=-20y$ C. $x^2=20y$ D. $x=20y^2$

18) What is the eccentricity of a conic $x^2-y^2=9$

    A. $\dfrac{2\sqrt{2}}{3}$ B. $\sqrt{2}$ C. $\dfrac{\sqrt{2}}{3}$ D. 0

19) For what value of q, vector $[1,-3,4]$ is parallel to vector $[q,9,-12]$

    A. $\sqrt{-199}$ B. $-\dfrac{1}{3}$ C. -3 D. 75

20) What is the projection of $\underline{i}-\underline{k}$ along $\underline{j}+\underline{k}$

    A. $-\dfrac{1}{2}$ B. $\dfrac{1}{\sqrt{2}}$ C. -1 D. $-\dfrac{1}{\sqrt{2}}$

MATHEMATICS HSSC–II

Time allowed: 2:35 Hours   **Total Marks Sections B and C: 80**

NOTE: Attempt any ten parts from Section 'B' and any five questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Graph paper will be provided on Demand.

SECTION – B (Marks 40)

Q. 2 Attempt any TEN parts. All parts carry equal marks.   **(10 x 4 = 40)**

(i) Evaluate $\lim\limits_{x \to 0} \dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$, $x > 0$

(ii) If $f(x) = \begin{cases} x + 2 & , \ x \le -1 \\ c + 2 & , \ x > -1 \end{cases}$, find "$c$" so that $\lim\limits_{x \to -1} f(x)$ exists.

(iii) Prove that $y\dfrac{dy}{dx} + x = 0$ if $x = \dfrac{1 - t^2}{1 + t^2}$, $y = \dfrac{2t}{1 + t^2}$

(iv) If $y = a\cos(\ln x) + b\sin(\ln x)$, prove that $x^2\dfrac{d^2 y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

(v) Find $f'(x)$ if $f(x) = \dfrac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}$

(vi) Evaluate $\int \dfrac{\sqrt{2}}{\sin x + \cos x}\, dx$

(vii) Find the area between the curve $y = x(x - 1)(x + 1)$ and the x-axis.

(viii) The three points $A(7, -1), B(-2, 2)$ and $C(1, 4)$ are the consecutive vertices of a parallelogram. Find the fourth vertex.

(ix) Find the lines represented by $x^2 + 2xy\sec\alpha + y^2 = 0$. Also find measure of the angle between them.

(x) Show that the lines $3x - 2y = 0$ and $2x + 3y - 13 = 0$ are tangents to the circle $x^2 + y^2 + 6x - 4y = 0$.

(xi) Find the focus, vertex and directrix of a parabola $(x - 1)^2 = 8(y + 2)$

(xii) An astroid has elliptic orbit with the sun at one focus. Its distance from the sun ranges from 17 million miles to 183 million miles. Write an equation of the orbit of the astroid.

(xiii) Find the area of a triangle with vertices $A(1, -1, 1)$, $B(2, 1, -1)$ and $C(-1, 1, 2)$.

(xiv) Find volume of the tetrahedron with the vertices $(2, 1, 8), (3, 2, 9), (2, 1, 4)$ and $(3, 3, 10)$

SECTION – C (Marks 40)

Note: Attempt any FIVE questions. All questions carry equal marks.   **(5 x 8 = 40)**

Q. 3 Prove that $\lim\limits_{x \to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$, where n is an integer and $a > 0$

Q. 4 Find the extreme values of a function defined as $f(x) = x^4 - 4x^2$

Q. 5 Show that $\int e^{ax}\sin bx\, dx = \dfrac{1}{\sqrt{a^2 + b^2}}\, e^{ax}\sin\left(bx - \tan^{-1}\dfrac{b}{a}\right) + C$

Q. 6 The vertices of a triangle are $A(-2, 3), B(-4, 1)$ and $C(3, 5)$. Find the centre of circumcircle of the triangle.

Q. 7 Minimize $z = 3x + y$; subject to the constraints: $3x + 5y \ge 15$; $x + 6y \ge 9$; $x \ge 0$; $y \ge 0$

Q. 8 Find the centre, foci, eccentricity, vertices and equations of directrices of a hyperbola $9x^2 - y^2 - 36x - 6y + 18 = 0$

Q. 9 Prove that angle in a semi-circle is a right angle.