UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2013**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

1 A random sample of 80 values of a variable X is taken and these values are summarised below.

$$n = 80 \qquad \Sigma x = 150.2 \qquad \Sigma x^2 = 820.24$$

Calculate unbiased estimates of the population mean and variance of X and hence find a 95% confidence interval for the population mean of X. [6]

2 A traffic officer notes the speeds of vehicles as they pass a certain point. In the past the mean of these speeds has been $62.3\,\text{km h}^{-1}$ and the standard deviation has been $10.4\,\text{km h}^{-1}$. A speed limit is introduced, and following this, the mean of the speeds of 75 randomly chosen vehicles passing the point is found to be $59.9\,\text{km h}^{-1}$.

(i) Making an assumption that should be stated, test at the 2% significance level whether the mean speed has decreased since the introduction of the speed limit. [6]

(ii) Explain whether it was necessary to use the Central Limit theorem in part **(i)**. [2]

3 The waiting time, T weeks, for a particular operation at a hospital has probability density function given by

$$\text{f}(t) = \begin{cases} \frac{1}{2500}(100t - t^3) & 0 \leqslant t \leqslant 10, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Given that $\text{E}(T) = \frac{16}{3}$, find $\text{Var}(T)$. [3]

(ii) 10% of patients have to wait more than n weeks for their operation. Find the value of n, giving your answer correct to the nearest integer. [5]

4 Goals scored by Femchester United occur at random with a constant average of 1.2 goals per match. Goals scored against Femchester United occur independently and at random with a constant average of 0.9 goals per match.

(i) Find the probability that in a randomly chosen match involving Femchester,

(a) a total of 3 goals are scored, [2]

(b) a total of 3 goals are scored and Femchester wins. [3]

The manager promises the Femchester players a bonus if they score at least 35 goals in the next 25 matches.

(ii) Find the probability that the players receive the bonus. [4]

5 A fair six-sided die has faces numbered 1, 2, 3, 4, 5, 6. The score on one throw is denoted by X.

(i) Write down the value of $\mathrm{E}(X)$ and show that $\mathrm{Var}(X) = \frac{35}{12}$. [2]

Fayez has a six-sided die with faces numbered 1, 2, 3, 4, 5, 6. He suspects that it is biased so that when it is thrown it is more likely to show a low number than a high number. In order to test his suspicion, he plans to throw the die 50 times. If the mean score is less than 3 he will conclude that the die is biased.

(ii) Find the probability of a Type I error. [5]

(iii) With reference to this context, describe circumstances in which Fayez would make a Type II error. [2]

6 The lifetimes, in hours, of Longlive light bulbs and Enerlow light bulbs have the independent distributions $\mathrm{N}(1020, 45^2)$ and $\mathrm{N}(2800, 52^2)$ respectively.

(i) Find the probability that the total of the lifetimes of 5 randomly chosen Longlive bulbs is less than 5200 hours. [4]

(ii) Find the probability that the lifetime of a randomly chosen Enerlow bulb is at least 3 times that of a randomly chosen Longlive bulb. [6]

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