Quadratic Equations
Exercise 4.1

Solve the following equations by factorization:

1. $3x^2+4x+1=0$

Solution:

$3x^2+4x+1=0$

$3x^2+3x+x+1=0$

$3x(x+1)+1(x+1)=0$

$(3x+1)(x+1)=0$

$3x+1=0$ and $x+1=0$

$x=-\frac{1}{3}$ $\quad$ $x=-1$

solution set $=\{-\frac{1}{3},-1\}$

2. $x^2+7x+12=0$

Solution:

$x^2+7x+12=0$

$x^2+4x+3x+12=0$

$x(x+4)+3(x+4)=0$

$(x+3)(x+4)=0$

$x+3=0$ and $x+4=0$

$x=-3$ $\quad$ $x=-4$

solution set $=\{-3,-4\}$

3. $9x^2-12x-5=0$

Solution:

$9x^2-12x-5=0$

$9x^2-15x+3x-5=0$

$3x(3x-5)+1(3x-5)=0$

$(3x+1)(3x-5)=0$

$3x+1=0$ and $3x-5=0$

$3x=-1$ $\quad$ $3x=5$

$x=\frac{-1}{3}$ $\quad$ $x=\frac{5}{3}$

solution set $=\{\frac{-1}{3},\frac{5}{3}\}$

4. $x^2 - x = 2$

Solution:

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

$(x+1)(x-2) = 0$

$x+1 = 0$ and $x-2 = 0$

$x = -1$ $\quad$ $x = 2$

solution set = $\{-1, 2\}$

5. $x(x+7) = (2x-1)(x+4)$

Solution:

$x(x+7) = (2x-1)(x+4)$

$x^2 + 7x = 2x^2 + 8x - x - 4$

$-2x^2 - 8x + x + 4 + x^2 + 7x = 0$

$-x^2 + 4 = 0$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

solution set = $\{-2, 2\}$

6. $\frac{x}{x+1} + \frac{x+1}{x} = \frac{5}{2}; x \neq -1, 0$

Solution:

$\frac{x}{x+1} + \frac{x+1}{x} = \frac{5}{2}$

$\frac{x^2 + (x+1)(x+1)}{x(x+1)} = \frac{5}{2}$

$\frac{x^2 + x^2 + x + x + 1}{x^2 + x} = \frac{5}{2}$

$\frac{2x^2 + 2x + 1}{x^2 + x} = \frac{5}{2}$

$2(2x^2 + 2x + 1) = 5(x^2 + x)$

$4x^2 + 4x + 2 = 5x^2 + 5x$

$-x^2 - x + 2 = 0$

$x^2 + x - 2 = 0$

$x^2+2x-x-2=0$

$x(x+2)-1(x+2)=0$

$(x-1)(x+2)=0$

$x=1$ and $x=-2$

solution set = $\{-2,1\}$

7. $\frac{1}{x+1}+\frac{2}{x+2}=\frac{7}{x+5}; x\neq -1,-2,-5$

Solution:

$\frac{1}{x+1}+\frac{2}{x+2}=\frac{7}{x+5}$

$\frac{(x+2)+2(x+1)}{(x+1)(x+2)}=\frac{7}{x+5}$

$\frac{x+2+2x+2}{x^2+2x+x+2}=\frac{7}{x+5}$

$\frac{3x+4}{x^2+3x+2}=\frac{7}{x+5}$

$(x+5)(3x+4)=7(x^2+3x+2)$

$3x^2+4x+15x+20=7x^2+21x+14$

$-4x^2-2x+6=0$

$4x^2+2x-6=0$

$2(2x^2+x-3)=0$

$2x^2+x-3=0$

$2x^2+3x-2x-3=0$

$x(2x+3)-1(2x+3)=0$

$(x-1)(2x+3)=0$

$x-1=0$ and $2x+3=0$

$x=1 \qquad x=\frac{-3}{2}$

solution set = $\{\frac{-3}{2},1\}$

Solve the following equations by completing the square:

9. $x^2-2x-899=0$

Solution:

$x^2-2x-899=0$

$x^2-2x+1-899=1$

$x^2-2x+1=900$

$(x-1)^2=900$

$x-1=\pm 30$

$x=1\pm 30$

$x=1+30$ and $x=1-30$

$x=31$ $\quad x=-29$

solution set = $\{-29,31\}$

10. $x^2+4x-1085=0$

Solution:

$x^2+4x-1085=0$

$x^2+4x+4-1085=4$

$x^2+4x+4=1089$

$(x+2)^2=1089$

$x+2=\pm 33$

$x=-2\pm 33$

$x=-2+33$ and $x=-2-33$

$x=31$ $\quad x=-35$

solution set = $\{-35,31\}$

11. $x^2+6x-567=0$

Solution:

$x^2+6x-567=0$

$x^2+6x+9-567=9$

$x^2+6x+9=576$

$(x+3)^2=576$

$x+3=\pm 24$

$x=-3\pm 24$

$x=-3+24$ and $x=-3-24$

$x=21$ $\quad x=-27$

solution set = $\{-27,21\}$

12. $x^2-3x-648=0$

Solution:

$x^2-3x-648=0$

$x^2-3x+(\frac{3}{2})^2-648=(\frac{3}{2})^2$

$x^2-3x+(\frac{3}{2})^2=648+\frac{9}{4}$

$x^2-3x+(\frac{3}{2})^2=\frac{2601}{4}$

$(x-\frac{3}{2})^2=\frac{2601}{4}$

$x-\frac{3}{2}=\pm\frac{51}{2}$

$x=\frac{3}{2}\pm\frac{51}{2}$

$x=\frac{3}{2}+\frac{51}{2}$ and $x=\frac{3}{2}-\frac{51}{2}$

$x=\frac{54}{2}$ $\qquad$ $x=\frac{-48}{2}$

$x=27$ $\qquad$ $x=-24$

solution set = $\{-24,27\}$

13. $x^2-x-1806=0$

Solution:

$x^2-x-1806=0$

$x^2-x+(\frac{1}{2})^2-1806=(\frac{1}{2})^2$

$x^2-x+(\frac{1}{2})^2=1806+\frac{1}{4}$

$(x-\frac{1}{2})^2=\frac{7225}{4}$

$(x-\frac{1}{2})=\pm\frac{85}{2}$

$x=\frac{1}{2}\pm\frac{85}{2}$

$x=\frac{1}{2}+\frac{85}{2}$ and $x=\frac{1}{2}-\frac{85}{2}$

$x=\frac{86}{2}$ $\qquad$ $x=\frac{-84}{2}$

$x=43$ $\qquad$ $x=-42$

solution set = $\{-42, 43\}$

14. $2x^2 + 12x - 110 = 0$

Solution:

$2x^2 + 12x - 110 = 0$

$2(x^2 + 6x - 55) = 0$

$x^2 + 6x - 55 = 0$

$x^2 + 6x + 9 - 55 = 9$

$(x+3)^2 = 55 + 9$

$(x+3)^2 = 64$

$x + 3 = \pm 8$

$x = -3 \pm 8$

$x = -3 + 8$ and $x = -3 - 8$

$x = 5$ $\qquad$ $x = -11$

solution set = $\{-11, 5\}$

Find roots of the following equations by using Quadratic Formula:

15. $5x^2 - 13x + 6 = 0$

Solution:

Compare it with Quadratic Equation: $ax^2 + bx + c = 0$

Here, $a = 5$, $b = -13$ and $c = 6$

Use Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(5)(6)}}{2(5)}$$

$$x = \frac{13 \pm \sqrt{169 - 120}}{10}$$

$$x = \frac{13 \pm \sqrt{49}}{10}$$

$$x = \frac{13 \pm 7}{10}$$

$x = \dfrac{13+7}{10}$ and $x = \dfrac{13-7}{10}$

$x = \dfrac{20}{10}$ $\qquad$ $x = \dfrac{6}{10}$

$x = 2$ $\qquad$ $x = \dfrac{3}{5}$

solution set = $\{2, \frac{3}{5}\}$

16. $4x^2 + 7x - 1 = 0$

Solution:

Compare it with: $ax^2 + bx + c = 0$

Use Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here, $a = 4$, $b = 7$ and $c = -1$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(4)(-1)}}{2(4)}$$

$$x = \frac{-7 \pm \sqrt{49 + 16}}{8}$$

$$x = \frac{-7 \pm \sqrt{65}}{8}$$

$x = \frac{-7 + \sqrt{65}}{8}$ and $x = \frac{-7 - \sqrt{65}}{8}$

solution set = $\{\frac{-7 - \sqrt{65}}{8}, \frac{-7 + \sqrt{65}}{8}\}$

17. $15x^2 + 2ax - a^2 = 0$

Solution:

Compare it with: $ax^2 + bx + c = 0$

Use Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here, $a = 15$, $b = 2a$ and $c = a^2$

$$x = \frac{-(2a) \pm \sqrt{(2a)^2 - 4(15)(-a^2)}}{2(15)}$$

$$x = \frac{-2a \pm \sqrt{4a^2 + 60a^2}}{30}$$

$$x = \frac{-2a \pm \sqrt{64a^2}}{30}$$

$$x = \frac{-2a \pm 8a}{30}$$

$x = \frac{-2a + 8a}{30}$ and $x = \frac{-2a - 8a}{30}$

$x = \frac{6a}{30}$ $\qquad$ $x = \frac{-10a}{30}$

$x = \frac{a}{5}$ $\qquad\qquad$ $x = \frac{-a}{3}$

solution set = $\{\frac{-a}{3}, \frac{a}{5}\}$

18. $16x^2 + 8x + 1 = 0$

Solution:

Compare it with: $ax^2 + bx + c = 0$

Use Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here, $a = 16$, $b = 8$ and $c = 1$

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(16)(1)}}{2(16)}$$

$$x = \frac{-8 \pm \sqrt{64 - 64}}{32}$$

$$x = \frac{-8 \pm \sqrt{0}}{32}$$

$$x = \frac{-8}{32}$$

$$x = \frac{-1}{4}$$

solution set = $\{\frac{-1}{4}\}$

19. $(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$

Solution:

$(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$

$x^2 - bx - ax + ab + x^2 - cx - bx + bc + x^2 - ax - cx + ac = 0$

$3x^2 - 2bx - 2ax - 2cx + ab + bc + ac = 0$

$3x^2 - 2(a+b+c)x + ab + bc + ac = 0$

Compare it with: $ax^2 + bx + c = 0$

Use Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here, $a = 3$, $b = -2(a+b+c)$ and $c = ab + bc + ac$

$$x = \frac{-(-2(a+b+c)) \pm \sqrt{(-2(a+b+c))^2 - 4(3)(ab+bc+ac)}}{2(3)}$$

$$x = \frac{2(a+b+c) \pm \sqrt{4(a^2+b^2+c^2+2ab+2bc+2ac) - 12ab - 12bc - 12ac}}{6}$$

$$x=\frac{2a+2b+2c\pm\sqrt{4a^2+4b^2+4c^2+8ab+8bc+8ac-12ab-12bc-12ac}}{6}$$

$$x=\frac{2(a+b+c)\pm\sqrt{4a^2+4b^2+4c^2-4ab-4bc-4ac}}{6}$$

$$x=\frac{2(a+b+c)\pm\sqrt{4(a^2+b^2+c^2-ab-bc-ac)}}{6}$$

$$x=\frac{2(a+b+c)\pm 2\sqrt{(a^2+b^2+c^2-ab-bc-ac)}}{6}$$

$$x=\frac{2\{(a+b+c)\pm 1\}\sqrt{(a^2+b^2+c^2-ab-bc-ac)}}{6}$$

$$x=\frac{(a+b+c)\pm 1\sqrt{(a^2+b^2+c^2-ab-bc-ac)}}{3}$$

$$x=\frac{(a+b+c)\pm\sqrt{(a^2+b^2+c^2-ab-bc-ac)}}{3}$$ Answer

20. $(a+b)x^2+(a+2b+c)x+b+c=0$

Solution:

$(a+b)x^2+(a+2b+c)x+b+c=0$

Compare it with: $ax^2+bx+c=0$

Use Quadratic formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Here, $a=a+b$, $b=a+2b+c$ and $c=b+c$

$$x=\frac{-(a+2b+c)\pm\sqrt{(a+2b+c)^2-4(a+b)(b+c)}}{2(a+b)}$$

$$x=\frac{-(a+2b+c)\pm\sqrt{a^2+4b^2+c^2+4ab+4bc+2ac-4ab-4ac-4b^2-4bc}}{2a+2b}$$

$$x=\frac{-(a+2b+c)\pm\sqrt{a^2+c^2-2ac}}{2a+2b}$$

$$x=\frac{-(a+2b+c)\pm\sqrt{(a-c)^2}}{2a+2b}$$

$$x=\frac{-a-2b-c\pm(a-c)}{2a+2b}$$

$$x=\frac{-a-2b-c+a-c}{2a+2b} \text{ and } x=\frac{-a-2b-c-a+c}{2a+2b}$$

$$x=\frac{-2b-2c}{2a+2b} \qquad x=\frac{-2a-2b}{2a+2b}$$

$x = \dfrac{-2(b+c)}{2(a+b)}$ $\qquad$ $x = \dfrac{-2(a+b)}{2(a+b)}$

$x = -\dfrac{(b+c)}{(a+b)}$ $\qquad$ $x = -1$

solution set $= \{-\dfrac{(b+c)}{(a+b)}, -1\}$